3.3 Exercises

In Exercises 1 and 2, use a graphing utility to graph the two functions in the same viewing window. What do the graphs suggest? Explain your reasoning.

1. $f(x) = \log_{10} x$, $g(x) = \dfrac{\ln x}{\ln 10}$

2. $f(x) = \ln x$, $g(x) = \dfrac{\log_{10} x}{\log_{10} e}$

In Exercises 3–10, evaluate the logarithm using the change-of-base formula. Round your result to three decimal places.

3. $\log_3 7$
4. $\log_7 4$
5. $\log_{1/2} 4$
6. $\log_{1/8} 64$
7. $\log_9(0.8)$
8. $\log_{1/3}(0.015)$
9. $\log_{15} 1460$
10. $\log_{20} 135$

In Exercises 11–18, rewrite the logarithm as a multiple of (a) a common logarithm and (b) a natural logarithm.

11. $\log_5 x$
12. $\log_3 x$
13. $\log_{1/5} x$
14. $\log_{1/3} x$
15. $\log_x \frac{3}{10}$
16. $\log_x \frac{3}{4}$
17. $\log_{2.6} x$
18. $\log_{7.1} x$

In Exercises 19–26, use the change-of-base formula to rewrite the logarithm as a multiple of a logarithm. Then use a graphing utility to sketch the graph.

19. $f(x) = \log_2 x$
20. $f(x) = \log_4 x$
21. $f(x) = \log_{1/2} x$
22. $f(x) = \log_{1/4} x$
23. $f(x) = \log_{11.8} x$
24. $f(x) = \log_{12.4} x$
25. $f(x) = \log_3 x^{1/2}$
26. $f(x) = \log_5 \dfrac{x}{3}$

In Exercises 27–46, use the properties of logarithms to write the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)

27. $\log_{10} 5x$
28. $\log_{10} 10z$
29. $\log_{10} \dfrac{5}{x}$
30. $\log_{10} \dfrac{y}{2}$
31. $\log_8 x^4$
32. $\log_6 z^{-3}$
33. $\ln \sqrt{z}$
34. $\ln \sqrt[3]{t}$
35. $\ln xyz$
36. $\ln \dfrac{xy}{z}$
37. $\ln \sqrt{a-1}, \quad a > 1$
38. $\ln\left(\dfrac{x^2-1}{x^3}\right), \quad x > 1$
39. $\ln z(z-1)^2, \quad z > 1$
40. $\ln \sqrt{\dfrac{x^2}{y^3}}$
41. $\ln \sqrt[3]{\dfrac{x}{y}}$
42. $\ln \dfrac{x}{\sqrt{x^2+1}}$
43. $\ln \dfrac{x^4\sqrt{y}}{z^5}$
44. $\ln \sqrt{x^2(x+2)}$
45. $\log_b \dfrac{x^2}{y^2z^3}$
46. $\log_b \dfrac{\sqrt{x}y^4}{z^4}$

Graphical Analysis **In Exercises 47 and 48, use a graphing utility to graph the two equations in the same viewing window. What do the graphs suggest? Explain your reasoning.**

47. $y_1 = \ln[x^3(x+4)]$, $y_2 = 3\ln x + \ln(x+4)$
48. $y_1 = \ln\left(\dfrac{\sqrt{x}}{x-2}\right)$, $y_2 = \frac{1}{2}\ln x - \ln(x-2)$

In Exercises 49–68, write the expression as the logarithm of a single quantity.

49. $\ln x + \ln 4$
50. $\ln y + \ln z$
51. $\log_4 z - \log_4 y$
52. $\log_5 8 - \log_5 t$
53. $2\log_2(x+3)$
54. $-6\log_6 2x$
55. $\frac{1}{3}\log_3 7x$
56. $\frac{5}{2}\log_7(z-4)$
57. $\ln x - 3\ln(x+1)$
58. $2\ln 8 + 5\ln z$
59. $\ln(x-2) - \ln(x+2)$
60. $3\ln x + 2\ln y - 4\ln z$
61. $\ln x - 2[\ln(x+2) + \ln(x-2)]$
62. $4[\ln z + \ln(z+5)] - 2\ln(z-5)$
63. $\frac{1}{3}[2\ln(x+3) + \ln x - \ln(x^2-1)]$
64. $2[\ln x - \ln(x+1) - \ln(x-1)]$
65. $\frac{1}{3}[\ln y + 2\ln(y+4)] - \ln(y-1)$
66. $\frac{1}{2}[\ln(x+1) + 2\ln(x-1)] + 3\ln x$
67. $2\ln 3 - \frac{1}{2}\ln(x^2+1)$
68. $\frac{3}{2}\ln 5t^6 - \frac{3}{4}\ln t^4$